Indirect Estimates of the

SOLIDS-NOT-FAT CONTENT OF MILK

The Basis for, and History of, Published Methods and Equations

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SUMMARY

A method of estimating the percentage of solids-not-fat in milk from the percentage of fat and the specific gravity was published as long ago as 1879, and three of the principal formulas were published before 1900. Most early formulas rested heavily on deductive logic. Basic values for the specific gravity of fat and solids-not-fat were arrived at by various means, and by algebraic calculations these values were converted into coefficients for estimating the percentage of solids-not-fat from the percentage of fat and specific gravity. More recently an inductive method--derivation of the required coefficients by the method of least squares--has been used, which affords somewhat greater precision and more convenient measures of expected differences between the estimated percentages of solids-not-fat and percentages determined by analysis.

More than 70 formulas for estimating solids-not-fat from fat alone, or from fat and specific gravity are given in the literature. The formulas are derived algebraically from two basic equations. The equations are linear. There is evidence to suggest that a nonlinear function gives a closer representation of the relationships, but the improvement is not substantial.

The precision of estimates of solids-not-fat is limited by (1) accuracy of the methods of analysis, (2) variability in the composition of milk resulting from the individuality of the cow and changes in her environment, and (3) variability in the specific gravity of the constituents of milk. The first two sets of factors offer most hope of progress toward greater precision through further research.

The standard errors of estimate of indirect methods of estimating the solids-not-fat of the mixed milk of a herd fall in a range of about 0.3 to 0.4 percent solids-not-fat when based on fat percentage, and about 0.1 to 0.35 when based on fat and specific gravity.

INDIRECT ESTIMATES OF THE SOLIDS-NOT-FAT CONTENT OF MILK 1/

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INTRODUCTION

The quantity of solids-not-fat in milk directly affects the yield of dairy products which may be obtained from it. Consequently, it has been proposed that producers should be paid for milk on the basis of its content of solids-not-fat (63, 123) 2/. The practicability of such proposals depends on knowing the solids-not-fat content of each producer's milk.

Regulations to protect consumers against adulterated milk contain specified minimum percentages of solids-not-fat in milk. This is another reason for having a rapid method for measuring or estimating solids-not-fat.

DERIVING FORMULAS

Equations representing the relationships among solids-not-fat content, fat content, and specific gravity of milk were first developed as long ago as 1879 (15). New equations have appeared from time to time, some as recent as 1950 (99) and 1951 (204).

The methods used in deriving these equations have been both deductive and inductive. In the deductive approach, it was postulated that the specific gravity of milk was a resultant of the percentages of fat and of solids-not-fat, both of whose specific gravities were constant or practically constant. The average specific gravities of the two components were determined, and from these constants were derived the coefficients of equations relating the percentages of fat and solids-not-fat and the specific gravity of milk. It should be noted that these equations were "reversible"; they were equally valid when either of the three quantities was unknown. However, no statement of probability could be attached to the estimated values of the unknown.

The inductive method of deriving the desired formulas consisted of determining the fat, solids-not-fat, and specific gravity values for a

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2/ Underscored numbers in parentheses refer to Literature Cited, p. 27.

number of samples, and from these determining the least squares regression equation. By this procedure, a precise measure of relationships resulted, and one to which a statement of probability could be attached concerning the estimated values of the variable that was treated as the unknown, or dependent variable in the equation. However, separate solutions, not algebraically identical, would be required for each variable for which an estimate might be wanted.

The importance of a statement of probability appears to have been recognized by some investigators, but many have ignored the notion, or have considered it unimportant. There has been a tendency to emphasize average or absolute values, particularly where the approach has been primarily deductive, and to ignore the size and frequency of differences between results of estimates and of analyses. In part, of course, this reflects the slow spread of familiarity with the appropriate statistical methods.

Fleischmann, Richmond, and Babcock

The voluminous literature on the subject of relationships between solids-not-fat and specific gravity of milk appears to begin with a report by Behrend and Morgen in 1879 (15). Demichel (50) cited the use of the principle involved in determining the solids content of a solution from the specific gravity of the solution as having been applied to the analysis of beet juice for its sugar content as early as 1830 and to other solutions in 1861 and 1877. Kaiser (103) in 1877 suggested the application of this principle to the examination of milk. Behrend and Morgen are usually conceded to have been the first to publish results of experiments applying this principle to the estimation of solids in milk. Within a few years, their work had been amplified and refined by Fleischmann and Morgen (61) and by Fleischmann (57). The formula published by Fleischmann in 1885 is one of the three most widely known formulas for estimating total solids of milk from the fat content and specific gravity. Fleischmann asserted his awareness that the formula would have only the status of a useful approximation (59). Others were more enthusiastic over formulas /Sobbe, (172)/ but Reiss (145) stressed the extent to which milk composition varied from the rule basic to the Fleischmann formula.

About the time Fleischmann published his first paper, numerous other authors made contributions on the subject, proposing formulas which differed in some respects from the formula of Fleischmann. It would be difficult to distinguish the part of each author's contribution that was original from the part that may have been borrowed from an earlier source--perhaps from some source not included in the known literature. One of these lines of development was through Hehner (75), and Hehner and Richmond (76). Richmond in 1895 proposed a formula which ranks with that of Fleischmann as one of the most widely used formulas for estimating the solids in milk (153).

The third formula that is important because of its widespread use was developed by Babcock, who in 1892 published two formulas differing in form (5). Babcock subsequently revised both formulas, one in 1896 (6) and the other in 1897 (7).

Other Formulas

Although the Fleischmann, Richmond, and Babcock formulas are the ones that appear most frequently in text books and manuals dealing with the estimation of solids-not-fat in milk, other formulas are referred to by various workers and deserve to be placed in some perspective in this field. In 1879 Clausnizer and Mayer (32) proposed a formula for estimating fat from the total solids and specific gravity of milk. At that time, such a formula was of practical importance because the methods for analyzing milk for its fat content were not as rapid, convenient, or precise as those developed later. It was then more convenient to determine the total solids than to determine the fat content of milk. (From the earliest of these dates, of course, specific gravity has been determined readily by means of lactometers, pycnometers, or Westphal balances.)

The formula of Clausnizer and Mayer rested on the same physical relationships as did that of Behrend and Morgen, and the Fleischmann formula was influenced by both. The Clausnizer and Mayer formula was adapted by Halenke and Möslinger (71) to the estimation of total solids. Ambühl (1) further modified the formula published by Halenke and Möslinger and gave it in a simplified form in his manual. Subsequent writers taking their material from one of these three sources did not appear always to have been aware of the linkage among them. Möslinger also contributed to a variant formula through correspondence with Barth who communicated the Möslinger formula to Bourcar (21).

Quesneville (140, 141, 142), in 1884 appears to have ante-dated Fleischmann in publishing a formula, but, probably because the constants in Quesneville's equation gave somewhat less acceptable results than Fleischmann's formula, Quesneville's paper did not exercise much influence on the subsequent literature. Quesneville mentions the works of Clausnizer and Mayer, and Hehner.

Giribaldo and Peluffo (66) argued that the form of the Quesneville formula was better justified theoretically and practically than was the form proposed by Fleischmann and Morgen. Although Giribaldo and Peluffo erred in this, they did contribute something to the understanding of these formulas by showing how specific gravities of both fat and solids-not-fat were implicit in all formulas. They compared the various formulas on the basis of these values. Giribaldo and Peluffo also suggested a variant formula applicable to milk from cows in Uruguay. The data and derivation of this formula were published in 1909 but nearly identical articles containing no new material were published by the same authors in 1919 (67) and again in 1941 (68).

The work of Quesneville inspired Pierre (135, 136, 137) to publish a note on formulas for estimating total solids of milk. Pierre neglected to credit his source in his original article but did so subsequently when Steinmann (175, 176) accused him of having made no contribution beyond that of Fleischmann. Pierre's formula was tested by Ranwez (143).

Reisz (146) and Bowditch and Bosworth (23) are reported to have published formulas, but the original articles were not seen, and the source in which they were cited did not give the formulas. Vieth (185) also published an article of some significance which was not available for review for this report.

Specific Gravity of Milk Fat

Differences in views concerning the specific gravity of fat account for many of the differences among formulas. Fleischmann derived his formula from a value of the specific gravity of fat which he believed to be representative or average, and from 1,004 analyses of milk for fat, total solids, and specific gravity. Many writers who proposed variations of Fleischmann's formula or who offered formulas of their own used different values for the specific gravity of the fat or solids-not-fat or both. In many instances, writers do not give details as to the methods used in determining the values for specific gravity of fat or solids-not-fat which they preferred (16, 50, 57, 189, 195).

Hehner (75), Hehner and Richmond (76), and Bakke and Honegger (10) describe carefully their methods for determining these specific gravity values. The latter authors and Hoyt et al. (92), Sharp and Hart (168), Janse (99), and Heinemann et al. (78), and Watson (196) developed modified methods of lactometry specially adapted to avoid the "Recknagle effect," the gradual increase in density of milk upon standing. Wunderlich (202) and Hawley (74) proposed special lactometric procedures for tropical conditions. Some other writers, such as Giribaldo and Peluffo (66, 67, 68), stress particularly their expectation of different values for specific gravity under conditions in their countries as a basis for modifying the formula.

Algebraic Derivation of the Formulas

The deductive methods used either for deriving the formulas or for supporting values based more largely on empirical methods were quite varied. They differ as to the assumptions with which they begin, and in the steps by which the formula is derived. In most instances, the derivation of the formula starts with the statement of relationship "the specific gravity of milk is a function of the fat, solids-not-fat and water present" (61). The equivalent of this statement was made in various ways by other authors. For example, "the volume of 100 grams of milk is a function of the volumes of fat and serum" (5). In another form, "the weight of 100 cubic centimeters of milk equals the weight of the volumes of fat and serum it

contains" (15). Sharp and Hart (168) refer to "the equation of additive specific volumes" in which the volume of a given weight of milk is equal to the percentages by weight of fat, solids-not-fat, and water divided by their respective specific gravities.

Many authors began with similar premises but proceeded by different routes to arrive at the form of final equation which they offer as a basis for estimating the solids-not-fat from the fat and specific gravity. Most of these can be shown to include the equivalents of equations (i) through (iv) below.

The volume of milk and its constituents can be expressed as ratios of the weight (100 grams of milk or percentage by weight of constituent parts) to the specific gravity of the milk or the respective constituent, thus:

$$\frac{100}{D} = \frac{F}{D_F} + \frac{N}{D_N} + \frac{W}{D_W} \quad 3/ \qquad \text{(i)}$$

The equation for this relationship was given as above, or with some rearrangement, by Babcock (5), Fleischmann and Morgen (61), Fleischmann (58), Richmond (155), Sharp and Hart (168), and Sommer (174). Some of the equivalent forms are:

$$D = \frac{100}{\frac{F}{D_F} + \frac{T-F}{D_N} + 100-T} \quad 4/ \qquad (61) \qquad \text{(ia)}$$

$$D = \frac{100\, D_N D_F}{D_N D_F\,(100-T) + D_N F + D_F\,(T-F)} \qquad (58) \qquad \text{(ib)}$$

$$\frac{100}{D} = \frac{F(D_S - D_F)}{D_F D_S} + \frac{100}{D_S} \quad 5/ \qquad (155) \qquad \text{(ic)}$$

From equation (i) it is desired to express N in terms of F and D. Accordingly, by substituting 100-F-N for W, and the numerical value of D_W; the equation is:

3/ The symbols represent:

D = specific gravity of milk; the weight of a given volume of milk at a given temperature relative to the weight of the same volume of water at a given (usually the same) temperature. The symbol "D" with a subscript denotes the specific gravity of the constituent identified by the subscript.

N = solids-not-fat in milk, percentage by weight.

F = fat in milk, percentage by weight.

W = water in milk, percentage by weight.

4/ T = total solids in milk, percentage by weight.

5/ S = serum in milk (100 grams of milk less the percentage of fat).

$$\frac{100}{D} = \frac{F}{D_F} + \frac{N}{D_N} + \frac{100-F-N}{1} \qquad (\underline{174}) \qquad \text{(ii)}$$

From this can be derived:

$$N = \frac{(\frac{1}{D_F} - 1)F}{(\frac{1}{D_N} - 1)} + \frac{1}{(\frac{1}{D_N} - 1)} \cdot \frac{100\ (D - 1)}{D} \qquad \text{(iii)}$$

Assuming D_F and D_N to be constants,

$$N = K_1F + K_2\ \frac{100\ (D-1)}{D} \qquad \text{(iv)}$$

Detailed derivations may be found in (5, 58, 61, 66-68, 75, 76, 117, 119, 155, 168, 174, 193, and 195).

A less detailed approach to an estimating equation involved the relationship between the changes in specific gravity of milk with changes in the percentage of serum. Quesneville (140-142), Babcock (5, 6), Hehner (75), and Hehner and Richmond (76) utilized this relationship as a basis for estimating equations.

Many authors have reported formulas for relating the solids-not-fat content of milk to the fat or to fat and specific gravity, the constants of which were calculated from observed data by the method of least squares. Among the earliest were Richmond (152) and Leonard (112). Anderson and Langmack (2), Cranfield et al. (39), Tocher (180), and Overman et al. (131, 133) reported results of such investigations in the 1920's. In the next decade, Anderson et al. (3), Kahlenberg and Voris (102), de Waal (193) and (195), Hawley (74), Janse (97), and Sharp and Hart (168) reported such determinations. More recently Bonnier et al. (20), Heinemann et al. (78), Provan (138), Janse (98), Richardson and Folger (149), and Jack et al. (94) derived formulas by this method. Van Dam (40, 41) urged the use of probable error rather than average difference as a measure of the accuracy of formulas.

Some authors have offered new formulas based on mean differences between solids-not-fat determined analytically and solids-not-fat calculated from an earlier formula. [Van Dam (42), Hoyt et al. (92), Boden and Campbell (18), and Ystgaard (203)]. Desai and Patel (51) reported that Richmond's formula generally gave greater values for solids-not-fat than did gravimetric methods.

A widely quoted equation for estimating solids-not-fat from the fat content of milk is that of Jacobson (95). He published what appears to be a free-hand regression based on the means of solids-not-fat values arrayed by fat tests. The values of the coefficients implicit in his data can be determined from coordinates of the extremes of the regression line. Other authors presenting relationships in the form of charts or tables without equations are Cranfield, et al. (39), Brown and Ekroth (27), Richardson (148) and the Milk Industry Foundation (122).

Literature reviews accompanying the more recent reports on fat and solids-not-fat relationships have been rather brief. Thorough coverage of the earliest literature may be found in Vieth (187), Fleischmann (59), Giribaldo and Peluffo (66-68), Koestler and Bakke (105, 106), Lüning (117), Richmond et al. (155), and Schoorl (165).

Types of Formulas

A summary of the various formulas or equations discloses that most of them fall into nine principal forms or types. Although these forms differ literally, most of them are algebraically identical as was shown by Sharp and Hart (168), Sommer (174), and others. Table 1 summarizes the principal features of about 70 formulas, as follows: Name of author, date of publication, country or region where the work was done, the temperatures specified for specific gravities, the coefficients of the formula, results calculated for milk having specified specific gravities and fat contents, and standard errors of estimate for a few of the formulas.

Greatest interest attaches to the Fleischmann (57), Richmond (153), and Babcock (7) formulas and those based on modified lactometry: Sharp and Hart (168) as variously modified (81, 115, 162, 203), Heinemann et al. (78), and Janse (99).

Testing the Validity of Formulas

The validity of the equations relating solids-not-fat to the fat, or to fat and specific gravity, has frequently been judged in relation to the average composition of milk samples from various sources. For example, Jack et al. (94) compared their results with the values for Jersey, Guernsey, and Holstein milk reported by Davis et al. (47). Numerous sources give average values of solids-not-fat corresponding to given fat percentages with or without specifying the regression equation that would represent the data and be useful for estimating solids-not-fat from the fat content of milk. The number of analyses included in such data varies from the 12 determinations reported by de Vries (189) to several thousand determinations reported by each of several authorities, including Bakalor (8), Overman et al. (132), and Nicholson and Lesser (129). A summary of the data from various sources appears in table 2.

Extensive data on the fat and solids-not-fat content of milk samples were published by Tobey (178) and Veale (184), but as they are not summarized it was not convenient to include them in table 2. Richardson and Folger (149) compared their data with those of Geiger (64) in addition to data shown in table 2.

A way of appraising the accuracy of formulas, favored by many investigators, was to apply several formulas to a given set of data. The formula showing the smallest average difference from gravimetrically determined values was presumed to be the best. Such comparisons were published by Van Dam (42), Farrington (56), Hehner (75), Höyberg (89, 90),

Table 1.—Equations relating the solids-not-fat content of milk to the fat content, or to fat and specific gravity

Author	Year	Country or region	Lactometry	Constants: a	Constants: b	Constants: k_1	Constants: k_2	Calculated value of N for: 1/ F=3.0 L=30.9	F=4.0 L=31.7	F=5.0 L=32.4	Standard error of estimate
Equations of type: N = aF + k 2/											
Anderson (3)	1931	Denmark	None	0.346		7.627		8.67	9.01	9.36	3/0.12
Heinemann, et al. (78)	1949	United States	"	0.3717		7.3073		8.42	8.79	9.17	0.327
Jack, et al. (94)	1951	California	"	0.444		7.07		8.40	8.85	9.29	0.36
Jacobson (95)	1936	New England	"	0.4		7.07		8.27	8.67	9.07	4/0.436
Janse (43, 98)	5/1929	Netherlands	"	0.534		6.55		8.15	8.69	9.22	
" "	6/1929	"	"	0.444		6.95		8.28	8.73	9.17	
" "	5/1949	"	"	0.231		7.96		8.65	8.88	9.12	
" "	6/1949	"	"	0.215		7.97		8.62	8.83	9.05	
Kahlenberg and Voris (102)	1931	Pennsylvania	"	0.783		6.187		8.54	9.32	10.10	
Provan (138)	1949	England	"	0.16		8.14		8.62	8.78	8.94	7/0.1
Tocher (180)	1925	Scotland	"	0.144		8.23		8.66	8.81	8.95	
Equations of type: N = aF + bL											
Ambuhl (1)	1899	Switzerland	15/15 C.	0.25	0.25			8.48	8.93	9.35	
Babcock (5)	1891	Wisconsin	60/60 F.	0.25	0.25			8.48	8.93	9.35	
" "	1897	"	60/60 F.	0.2	0.25			8.33	8.73	9.10	4/
Bouriez (22)	1921	France	15/15 C.	0.17	0.2666			—	—	—	
Burg (28)	—	—	8/	0.175	0.25			8.25	8.63	8.98	
Cini (31)	1933	—	—	0.2	0.2665			8.83	9.25	9.63	
Demichel (50)	1904	France	8/	0.14	0.2659			8.64	8.99	9.32	
Giribaldo and Peluffo (66)	1909	Uruguay	15/4 C.	0.19	0.282			—	—	—	
Hehner and Richmond (76)	1888	England	15.5/15.5 C.	0.164	0.254			8.34	8.71	9.05	
Janse (97)	1932	Netherlands	8/	0.216	0.247			8.28	8.69	9.08	
Pierre (135)	1904	France	15/15 C.	0.19	0.264			8.73	9.13	9.50	
Quesneville (140)	1884	France	15/15 C.	0.06	0.275			8.68	8.96	9.21	
Sharp and Hart (168)	1936	New York	30/30 C.	0.2648	0.2586			9/8.32	9/8.80	9/9.24	
de Waal (195)	1934	Netherlands	8/	0.3714	0.2285			8.17	8.73	9.26	

See footnotes at end of table.

Table 1.—Equations relating the solids-not-fat content of milk to the fat content, or to fat and specific gravity —Continued

Author	Year	Country or region	Lactometry	Constants: a	Constants: b	Constants: k_1	Constants: k_2	Calculated value of N for: 1/ F=3.0 L=30.9	F=4.0 L=31.7	F=5.0 L=32.4	Standard error of estimate
Equations of type: $N = aF + b(L + k)$											
Herrington (81)	1946	New York	30/60 C./F.	0.2648	0.2594	3.0		8.42	8.89	9.37	4/0.176
Livak (115)	1944	Pennsylvania	30/15	0.2648	0.2586	3.5		8.53	9.00	9.47	
Rueda (162)	1943	Wisconsin	30/15 C.	0.2648	0.2586	3.2		8.45	8.92	9.39	
Equations of type: $N = aF + bL + k$											
Bertschinger (11)	1907	—	8/	0.25	0.25	0.07		8.55	9.00	9.42	
Boden and Campbell (18)	1942	England	10/68/60 C.	0.21	0.25	0.66		—	—	—	
Bycichin (29)	1936	Czechoslovakia	15/15 C.	0.25	0.25	0.09		8.57	9.02	9.44	
Hälenke and Moslinger (71)	1886	Germany	—	0.25	0.25	0.07		8.55	9.00	9.42	
Heinemann, et al. (78)	1949	United States	30/15.5 C.	0.2566	0.2512	1.0467		8.45	8.91	9.36	0.174
Herz (84)	1901	Germany	8/	0.2	0.25	0.20		8.53	8.93	9.30	
Herz [cited by (165)]	—	—	8/	0.2	0.25	0.26		8.59	8.99	9.36	
Hoyt, et al. (92)	1928	California	15/15 C.	0.2	0.25	0.2		8.53	8.93	9.30	
Leonard (112)	1900	England	8/	0.26	0.26	-0.38		8.43	8.90	9.34	11/0.065
Overman, et al. (131)	1925	Illinois	15.5/15.5 C.	0.2	0.25	0.105		8.43	8.83	9.21	12/0.242
Overman, et al. (131)	1925	Illinois	15.5/15.5 C.	0.2	0.25	0.173		8.50	8.90	9.27	13/0.340
de Paulo (134)	1947	Brazil	15/15 C.	0.2	0.25	14/0.26		8.59	8.99	9.36	
Richmond (155)	1914	England	15.5/15.5 C.	0.2	0.25	0.14		8.47	8.87	9.24	
Tocher (180)	1925	Scotland	15.5/15.5 C.	0.229	0.251	0.003		8.45	8.88	9.28	
Equations of type: $N = \frac{aF + bL}{k}$											
Babcock (5)	1891	Wisconsin	60/60 F.	0.7	1.0	3.8		8.68	9.08	9.45	
Babcock (5)	1891	Wisconsin	60/60 F.	1.0	1.0	4.0		8.48	8.93	9.35	
Bourcar-Moslinger (21)	1889	Germany	15/15 C.	1.0	1.0	4.0		8.48	8.93	9.35	
Hälenke and Moslinger (71)	1886	Germany	15/15 C.	1.0	1.0	4.0		8.48	8.93	9.35	
Höyberg (89, 90, 91)	1913	Germany	—	1.0	1.0	4.0		8.48	8.93	9.35	
Leys (114)	1904	France	15/15 C.	1.0	1.0	4.0		8.48	8.93	9.35	

See footnotes at end of table.

Table 1.—Equations relating the solids-not-fat content of milk to the fat content, or to fat and specific gravity —Continued

Author	Year	Country or region	Lactometry	Constants: a	Constants: b	Constants: k_1	Constants: k_2	Calculated value of N for: 1/ F=3.0 L=30.9	F=4.0 L=31.7	F=5.0 L=32.4	Standard error of estimate
Equations of type: $N = \frac{aF + bL}{k_1} + k_2$											
Fleischmann (59)	1914	Germany	15/15 C.	0.8	1.0	4.0	0.25	8.58	8.98	9.35	
Janse (99)	1950	Netherlands	15/15 C.	1.0	1.0	4.0	0.45	8.93	9.38	9.80	
Muller-Hoessli (127)	1944	Switzerland	8/	0.8	1.0	4.0	0.26	8.59	8.99	9.36	
Equations of type: $N = aF + b\,\frac{100\,D - 100}{D}$											
Bém (16)	1933	Hungary	8/	0.2	2.663			8.58	8.98	9.36	
Codex Aliment. Neth. (34)	1907	Netherlands	15/15 C.	0.17	2.60			8.30	8.67	9.01	
Fleischmann (57)	1885	Germany	15/15 C.	0.2	2.665			8.59	8.99	9.36	
Hawley (74)	1933	India	85/60 F.	0.328	2.872			—	—	—	
Koestler and Bakke (105)	1923	France	15/15 C.	0.19	2.550			8.21	8.60	8.95	
Richmond (152)	1894	England	15.5/15.5 C.	0.2	2.625			8.47	8.87	9.24	
Sharp and Hart (168)	1936	New York	30/30 C.	0.2537	2.68			8.35	8.82	9.25	
de Vries (189)	1916	Netherlands	15/15 C.	0.23	2.56			8.36	8.79	9.18	
de Waal (193)	1931	Netherlands	8/	0.37	2.36			8.18	8.73	9.26	
Wunderlich (202)	1916	Neth. Indies	27.5/27.5 C.	0.23	2.71			9/8.35	9/8.79	9/9.20	
Equations of type: $N = aF + \frac{bL + k_1}{D} + k_2$											
Herrington (81)	1946	New York	30/60 C./F.	0.2537	0.268	3.0	—	8.44	8.89	9.35	
Livak (115)	1944	Pennsylvania	30/15 C.	0.2537	0.268	3.5	—	8.57	9.02	9.48	
Rueda (162)	1943	Wisconsin	30/15 C.	0.2537	0.2671	3.2	—	8.46	8.92	9.37	
Ystgaard, et al. (203)	1951	Iowa	30/15.5 C.	0.2537	0.268	3.0	-0.15	8.29	8.74	9.20	

See footnotes at end of table.

Table 1.—Equations relating the solids-not-fat content of milk to the fat content, or to fat and specific gravity
—Continued

Author	Year	Country or region	Lactometry	Equation	Calculated value of N for: 1/ F=3.0 L=30.9	F=4.0 L=31.7	F=5.0 L=32.4	Standard error of estimate
Miscellaneous equations								
Babcock (5)	1891	Wisconsin	60/60 F.	$N = \left(\frac{100 D - D F}{100 - 1.0753 D F} - 1\right)(100-F)\ 2.6$	8.67	9.09	9.49	
Babcock (6)	1895	Wisconsin	60/60 F.	$N = \left(\frac{100 D - D F}{100 - 1.0753 D F} - 1\right)(100-F)\ 2.5$	8.33	8.74	9.12	
Brown (25)	1886	New York	60/60 F.	$N = D - \left(\frac{1.000 - F\ 0.001}{0.00375}\right)$	9.04	9.52	9.97	
Fleischmann and Morgen (61)	1882	Germany	15/15 C.	$N = .173\ F + .271\left(100 - \frac{100}{D}\right)$	8.64	9.02	9.37	
Hehner and Richmond (76)	1888	England	15.5/15.5 C.	$N = \frac{F}{5} + \frac{L}{4}$	8.33	8.73	9.10	
Heinemann et al. (78)	1949	United States	30/15.5 C.	$N = .7070 + .1514\ F + .1993\ L + .6585P$	—	—	—	0.080
Janse (99)	1950	Netherlands	20/4 C.	$N = .23\ F + 2.6\ \frac{100\ (D - 0.9982)}{D}$	—	—	—	
Richmond (153)	1895	England	15.5/15.5 C.	$N = \frac{F}{5} + \frac{L}{4} + .14$	8.47	8.87	9.24	
Saar (163)	1928	Germany	8/	$N = .25L + .2F + 0.26 + C$ 15/	8.59	8.99	9.36	
de Waal (193)	1931	Netherlands	8/	$N = .208\ F + .206\ P + 2.43\ \frac{100D - 100}{D}$	—	—	—	

1/ The lactometer degrees shown in the column heading are for equations in which specific gravities are taken with milk and water both at 15°/15°. For equations in which lactometer readings are at 30°/15° C., (or 85°/60° F.) the appropriate values for L would be 26.4, 27.2 and 28.0, according to data of Heinemann et al. (44). In either case the lactometer readings are consistent with specific gravities of 0.93 and 1.6 for fat and solids-not-fat respectively, at 15° C., and with 9.0 percent solids-not-fat accompanying 4.0 percent fat, and changing at the rate of 0.4 percent solids-not-fat with each percent of fat.

2/ N = percent of solids-not-fat; F = percent of fat; T = total solids; L = lactometer degrees; D = specific gravity; P = protein; a, b, k, k_1, k_2, etc. are constants.

3/ Average deviation.

4/ Reported by Heinemann, et al. (78) as 0.209. From data published by the New York State Department of Markets (128) it was calculated to 0.182.

5/ Barn season.

6/ Pasture season.

Footnotes for Table 1 (continued)

7/ Approximate probable error when F = 3.45.
8/ Not specified, but evidently 15°/15° C.
9/ Values of L for this equation reduced by 1.8 in accord with relationships indicated in table 1 of Sharp and Hart (169).
10/ Heated to 40° C. for 5 minutes and cooled to 68° F.
11/ Probable error.
12/ Herd milk.
13/ Milk of individual cows.
14/ Use 1.0348 (.250) if D is less than 1.029.
15/ C = -0.02 when L = 21.7-23.7; -0.01 when L = 23.8-26.5; 0.00 when L = 26.6-38.4; and -0.01 when L = 38.4-40.0.

Table 2.—Average fat and solids-not-fat content of milk reported, or computed from data reported, by specified investigators

Investigator	Year	Country or State	Determinations	Herds	Cows	Lactations	Fat	Solids-not-fat
			No.	No.	No.	No.	Pct.	Pct.
Babcock (7)	1897	United States	—	—	—	—	3.75	9.05
Caulfield, et al. (30)	1939	Kansas	1,101	—	—	—	4.57	9.02
Collier (35)	1891	New York	930	—	14	—	4.24	9.40
Davis (49)	1947	Arizona	120	5	—	—	3.8	8.8
Eckles and Shaw (53)	1913	Missouri	—	—	11	—	3.88	8.78
Haecker (70)	1914	Minnesota	543	—	—	—	4.73	8.96
Hoyt, et al. (92)	1928	California	33	15	18	—	4.028	8.777
Kahlenberg and Voris (102)	1931	Pennsylvania	134	—	—	—	3.41	8.86
Livak (115)	1944	Pennsylvania	30	—	—	—	4.14	8.64
Lythgoe (119)	1914	Massachusetts	434	—	—	—	4.21	8.77
Moore and Morrow (126)	1936	New Hampshire	560	40	—	—	3.68	8.64
Overman, et al. (132)	1939	Illinois	2,426	—	—	—	4.142	9.248
Richardson and Folger (149)	1950	California	1,803	—	—	—	4.08	8.71
Shaw and Eckles (170)	1911	Missouri	—	—	—	16	3.97	8.92
Shaw and Fourt (169)	1936	Idaho	902	—	71	—	4.17	8.73
Sherman (171)	1906	New York	60	1	600	—	5.42	9.22
White and Judkins (197)	1918	Connecticut	—	—	49	126	4.13	8.84
Woodward and Lee (201)	1908	Louisiana	76	38	—	—	4.54	8.96
Anderson and Langmack (2)	1923	Denmark	—	30	—	—	3.52	8.84
Cranfield, et al. (39)	1927	England	732	15	—	—	3.712	8.746
Jones (101)	1935	England	—	—	—	—	3.72	8.89
Leonard (112)	1900	England	137	—	—	—	2.94	8.11
Nicholson and Lesser (129)	1934	England	5,450	2	115	—	3.65	8.42
Provan (138)	1949	England	1/	—	—	—	2/3.60	2/8.90
							3/3.53	3/8.60
Robertson (159)	1919	England	—	1	13	38	3.65	8.76
Vieth (188)	1889	England	15,227	—	—	—	3.81	9.13
Tocher (179)	1919	Scotland	384	—	—	—	3.495	8.85
Tocher (180)	1925	Scotland	676	—	676	—	3.953	8.804
Bockairy (17)	1904	France	—	—	875	—	3.98	8.64
Fleischmann (57)	1885	Germany	1,004	—	—	—	3.242	8.710
Höyberg (89, 90)	1913	Germany	20	—	—	—	3.31	8.90
Janse (98)	1950	Netherlands	—	—	—	—	4/3.58	4/8.52
							5/4.12	5/8.90
de Vries (189)	1916	Netherlands	—	—	—	—	3.14	8.16
Koestler and Bakke (105, 106)	1923	Switzerland	6/ 16	—	—	—	4.16	8.85
			6/ 19	—	—	—	3.83	8.65
Bakalor (8)	1948	South Africa	2,808	—	—	—	3.64	8.56

1/ Monthly analyses of bulk milk at creameries under the Milk Marketing Board.
2/ 1923-33.
3/ 1938-46.
4/ Data for year 1927-28.
5/ Data for year 1948-49.
6/ Each determination on a sample from a vat of 2,000 to 5,000 kilograms. The different sets of determinations were on samples from different plants.

Muller-Hoessli (127), Pierre (135, 136), Schoorl (165, 166), Shaw and Eckles (170), V. Sobbe (172), de Vries (189), Winton (199), and Woll (200). New York State Department of Agriculture and Markets (128) reported the differences between values calculated by the Babcock formula (6) and values determined gravimetrically. In 3,557 samples of known purity, the estimated percentage of solids-not-fat ranged from 1.24 percent less to 1.74 percent more than the gravimetrically determined values. The average difference was +0.097 percent, and the standard deviation of the difference was ±0.182 percent.

The rounding of coefficients for greater convenience explains some of the differences among the formulas in table 1. A succession of steps from a lengthy formula to one greatly simplified by rounding is given by Lüning (117). Of course, Babcock's formula N = .25L + .2F represents the simplest practical formula (6). An even simpler form, N = (L + F)/4, was proposed earlier by Babcock (5), and by Ambühl (1). Several investigators [Lüning (117), and Saar (163)] retained some of the information lost in rounding by adding a correction factor to the formula.

Linearity of Relationships

The great bulk of the literature describes the relationship between the solids-not-fat and fat as being linear. At least, it accepts the linear description as being of practical value. But Richardson and Folger (149) question the assumption of linearity. Their suggestion is to apply a series of linear regressions to different segments of the range of fat tests. The resulting zig-zag line of relationship may describe their data, but it is not convenient to apply in practice and it has no apparent logical basis. Reiss (145) also noted a zig-zag curve though his data were for only 24 cows.

In disputing the validity of a single linear relationship over the whole range of fat tests, Richardson and Folger (149) cite Tocher (180) whose "survey revealed a uniform rise in the average percentage of solids-not-fat with ascending values of percentage of butterfat; the butterfat and solids-not-fat could be represented, however, not by normal curves, but rather by Pearson's type IV curves." The latter part of this quotation describes the frequency distributions of the variables singly. It is irrelevant to the question of whether a linear regression describes well the relationship between them. The type IV curve referred to is more sharply peaked than the normal curve, but this is not inconsistent with the presence of linear relationships.

Richardson and Folger's citations of Brown and Ekroth (27) and the Milk Industry Foundation (122) are more to the point, but these sources did not offer any test of the significance of the departure of their curvilinear regressions from linearity. Bonnier, et al. (20) are cited as finding "no linear or quadratic relationship between the percentage of protein or lactose and that of fat except within intervals of 0.6 percent fat." The choice of words is inaccurate, on the part of both Richardson and Folger and Bonnier et al. The means of arrays of protein and lactose percentages within groups of fat percentages were found to deviate from a

linear regression by more than would be expected by chance. There is some doubt that their test was valid, but even if it were, a casual inspection of their results shows an insignificant gain in precision from using a more elaborate function, or the tabular method which they recommend. Jack et al. (94) found that quadratic and cubic regression equations gave a better fit than a linear equation, but that the standard errors of estimate were no different.

Cranfield et al. (39) found a parabolic relation between fat and solids-not-fat, the fat percentages falling from 3.8 to 3.6 as solids-not-fat rose from 8.0 to 8.8, and fat percentages rising from 3.6 to 3.8 as solids-not-fat rose from 8.8 to 9.2. The samples of morning and evening milk had been analyzed separately, and it was thought that this might account for the nature of the curve.

The question considered by Richardson and Folger (149) and by Bonnier et al. (20), was whether the regression coefficients determined separately for each segment of the range of fat percentages were sufficiently alike to indicate that the same inferences could be drawn from a single coefficient as from separate coefficients for each segment of the universe from which the sample was drawn. But another question of greater practical meaning concerns the accuracy with which the solids-not-fat content of a sample can be predicted. As was shown by Jack et al., the standard error of estimate was not improved by using equations of higher degrees.

Richardson and Folger and Bonnier et al. did not report any measures of the errors of estimate. Conceivably, a linear regression could be used to represent the data, but with such a wide range of deviations from the regression that the estimate would not be sufficiently practical for such uses as paying producers, establishing a presumption of adulteration, or controlling the composition or yield of dairy products. On the other hand, one may be confident that the regression equation offered by Jack et al. (94) when used in California will give, in two samples out of three, the solids-not-fat content from the fat content within 0.36 percent of the value that would be obtained gravimetrically by the official method of the Association of Official Agricultural Chemists, and that 19 out of 20 monthly composite samples would be within 0.1 percent of the gravimetric value.

There is some analogy between the problem of choosing an equation of suitable form and the problem of choosing suitable variables. Thus, Heinemann et al. (78) found that the standard error of estimate of solids-not-fat from fat alone was 0.327, but that the addition of lactometer readings gave an equation with a standard error of estimate of 0.174 percent. The prospective user of such equations thus has some basis for choosing between degrees of accuracy and degrees of simplicity.

For a portion of the samples analyzed by Heinemann et al., fat percentages and lactometer readings gave the solids-not-fat with a standard error of estimate of 0.138 percent, and the addition of protein percentage

reduced the standard error of estimate to 0.080 percent. De Waal (193) also suggested an equation relating total solids to fat, specific gravity, and protein.

FACTORS INFLUENCING THE RELATIONSHIP BETWEEN THE FAT AND SOLIDS-NOT-FAT

The factors influencing the relationship between the fat and solids-not-fat of milk are numerous and they have been widely studied. It is pertinent to take note of them here because the variations to which they give rise are beyond the power of any simple equation to correct. Some of the factors may be controlled, but in most circumstances there will remain a substantial number of uncontrolled factors, giving rise to variation which will influence the standard error of estimate.

Methods of Analysis

The factors affecting fat and solids-not-fat relationships which are most subject to control are those having to do with the methods of determining the fat and solids-not-fat content and the specific gravity of a sample of milk. No method for any of these factors has been so satisfactory as to be accepted without question. In the order of their precision, there is probably greatest general acceptance of the butterfat tests, yet the widely accepted Babcock test has been criticized because of the variability of its results (80).

Gravimetric methods are frequently considered to be the standard against which estimates of total solids or solids-not-fat from fat and lactometer readings should be judged. But Boden and Campbell (18), Heinemann et al. (78), and Ystgaard et al. (204) have called attention to the fallibility of results from gravimetric methods for total solids. Heinemann et al. (78) suggested that the error of estimate of total solids from fat and lactometer readings is not greater than the error of gravimetric methods, and Sharp and Hart (168) offered evidence that errors in the determination of the solids in whole milk by drying actually exceed the errors in determination of the solids in whole milk by calculations from the specific gravity and the fat content.

Ystgaard et al. (204) observed that the official method of the Association of Official Agricultural Chemists gave higher, and for that reason presumably more satisfactory, results for total solids than another method but that the other method gave results which varied less among replicate determinations.

Richmond (155) conceded that Fleischmann's formula was satisfactory if the fat percentage were determined by the Soxhlet method.

Although considerable confidence may be had in specific gravity measurements as a basis for estimating the solids-not-fat content of milk, their accuracy is strongly dependent on the determination of specific

gravity at the temperature appropriate to the equation being used, and on the previous history of the sample. A considerable difference between the specific gravity of fat in the liquid state and in the solid state, and the fact that the fat is liquid at the temperature at which it is drawn from the cow and solid at the temperature specified for most lactometric procedures, are factors which have considerable effect on the validity of estimating equations. Boden and Campbell (18) report a difference of approximately 1.3 lactometer degrees between determinations at 60° and 68° F. and Heinemann et al. (78) found a difference of about 4.55 lactometer degrees between determinations at 15 1/2° and 30° C. Any formula in which specific gravity is a factor should be applied only to data obtained by the same method of lactometry as was used in deriving the equation.

The most widely used temperatures have been 15°/15° C., 15.5°/15.5° C. and 60°/60° F. which are nearly identical. As has been mentioned above, the fat is solid at this temperature but it becomes solid only at 12 to 15 hours after milking. The slow rate of contraction of the butterfat during this interval, gives rise to the "Recknagle effect," and has complicated the problem of measuring the specific gravity of milk (62). Most of the early writers were well aware of this phenomenon, and specified clearly that specific gravity was to be measured only when the "terminal value" had been reached (71, 150). In order to permit accurate measurement of the specific gravity as soon as possible after milking, various authors have suggested using the specific gravity measured at temperatures above the melting point of the milk fat. Bakke and Honegger (10) first suggested that the repeatability of specific gravity determinations could be increased by heating the sample to 40° C. and cooling it immediately to 15° before determining the specific gravity. Subsequent studies have been cited above (p. 4). The recent work by Watson (196) promises much greater precision than any other method yet developed. From unpublished data furnished by Watson, a deductive type equation gave differences from gravimetric values with a standard deviation of 0.068. An equation derived by the least squares method had a standard error of estimate of 0.061. These are much closer than the 0.1 to 0.25 for standard errors shown for various equations in table 1.

The relationship between fat and solids-not-fat has been studied by some authors using specific gravity methods for estimating the solids-not fat. Inasmuch as most equations have tended to give slightly different results when used on other data than those from which they were derived, conclusions based on such studies should be examined carefully. It may be noted particularly that the differences between calculated and gravimetric values of solids-not-fat have been found to change seasonally (13, 18, 112, 164).

Individuality of the Cow

The basic sources of variation in the solids-not-fat content of milk are in the genetic and physiologic attributes of the cow herself. Numerous reports have established differences between breeds as to the

average content of solids-not-fat and as to the relationship between fat percentage and solids-not-fat percentage. Data pertaining to this factor appear in Bockairy (17), Caulfield et al. (30), Cranfield (36), Davis et al. (47, 48, 49), Huyguen (93), Jack et al. (94), Lythgoe (119), Overman et al. (132, 133), Provan and Jenkins (139), Richardson and Folger (149), Rowland (160, 161), Schutte (167), Shaw and Fourt (169), Tocher (181), White and Judkins (197), Collier (35), Eckles and Shaw (53), Bém (16), and Shaw and Eckles (170). Data from some of these sources are shown in table 3.

Long-time changes in the composition of milk in England are reported by Provan (138, 139) and Kay (104). Here the solids-not-fat declined. In the Netherlands, Janse (98) found an increase in both fat and solids-not-fat, the latter increasing at more than the usual relationship to fat. There appear to be inherited differences among families within breeds. Such have been noted by Moore and Keener (125), Provan and Jenkins (139), Van Rensberg (147), Richardson and Folger (149), Rowland (161), and Shaw and Fourt (169).

The solids-not-fat content of milk produced by any individual cow thus is partly determined by her breed and her ancestry within the breed, but there remains a considerable variation from milking to milking, from time to time throughout the lactation period, and from lactation to lactation which is random or in part ascribable to other factors. Some of these factors may be described as physiological and pathological.

During the lactation period the solids-not-fat tend to fall for some weeks after calving and then to rise to the end of the lactation period. This pattern has been reported by Bartlett (12), Davis et al. (48), Eckles and Shaw (52), Van Rensburg (147), Rowland (161), Tocher (180, 182), and White and Judkins (197). Cranfield (36, 37) also mentions period of lactation as a factor in the level of solids-not-fat in milk. Investigators differ as to the conformation of the pattern of solids-not-fat percentages during lactation, or even as to whether a pattern exists. Lythgoe (119) presented data showing that the solids-not-fat percentage increases from the first month. Robertson (159), on the other hand, states that the relationship of period of lactation to level of solids-not-fat is not clear. His data showed the solids-not-fat percentage to be lower in each of the last 3 months of lactation than in any of the first 3 months. Shaw and Fourt (169) found that the month of lactation was not a statistically significant factor in the level of solids-not-fat.

Whether the percentage of solids-not-fat will rise toward the end of the lactation period may be influenced by pregnancy. Bartlett (12) and Rowland (161) observed that a rise in the solids-not-fat occurs only if the cow is pregnant.

There is general agreement that the average level of solids-not-fat decreases with successive lactations. The solids-not-fat percentage is highest in the lactations following the first and second calvings. This

Table 3.--Average fat and solids-not-fat content of milk reported for specified breeds of cattle

Investigator	Holstein		Jersey		Guernsey		Ayrshire		Shorthorn	
	Fat	Solids-not-fat	Fat	Solids-not-fat	Fat	Solids-not-fat	Fat	Solids-not-fat	Fat	Solids-not-fat
	Percent	Percent	Percent	Percent	Percent	Percent	Percent	Percent	Percent	Percent
Bém (16)	3.27	8.80	4.91	9.69	--	--	--	--	4.52	9.43
Caulfield (30)	3.69	8.52	5.53	9.54	5.09	9.29	4.32	8.91	--	--
Collier (35)	3.46	9.07	5.61	9.80	5.12	9.47	3.57	9.35	--	--
Davis (48)	3.53	8.58	5.16	9.45	4.85	9.38	--	--	--	--
Eckles and Shaw (53)	3.09	8.29	4.95	9.14	--	--	3.68	8.73	3.73	8.96
Kahlenberg and Voris (102)	3.41	8.86	--	--	--	--	--	--	--	--
Lythgoe (119)	3.41	8.28	5.65	9.10	5.23	9.37	4.01	8.63	--	--
Nicholson and Lesser (129)	3.43	8.28	--	--	4.88	8.92	3.78	8.57	--	--
Overman et al. (132)	3.55	8.96	5.18	9.51	5.19	9.67	4.15	8.95	--	--
Provan and Jenkins (139)	3.45	8.58	4.41	8.93	--	--	3.72	8.73	3.65	8.68
Richardson and Folger (149)	3.63	8.44	5.55	9.54	--	--	3.96	8.74	--	--
Robertson (159)	3.65	8.76	--	--	--	--	--	--	--	--
Shaw and Fourt (169)	3.31	8.41	5.35	9.18	--	--	--	--	--	--
Sherman (171)	--	--	5.42	9.22	--	--	--	--	--	--
Tocher (181)	3.62	8.62	--	--	--	--	4.08	8.75	--	--
White and Judkins (197)	3.41	8.53	5.32	9.07	4.49	8.92	4.12	8.82	--	--

conclusion is supported by Bartlett (12), Cranfield (36), Van Rensburg (147), Rowland (161), Tocher (180, 182), and White and Judkins (197). The amount of decline from lactation to lactation is small, ranging from 0.03 to 0.18 percent per lactation (table 4).

Table 4.--Average solids-not-fat content of milk during successive lactations

Investigator	Lactation			
	First	Second	Third	Fourth
Bartlett (12)	9.49	9.32	9.22	--
Van Rensburg (147)	8.76	8.80	8.62	8.54
White and Judkins (197)	8.93	8.88	8.85	--

A lower level of solids-not-fat occurs in cows affected with mastitis /Richardson and Foger (149) and Rowland (161)/. Davies (44) examined milk containing an abnormally low percentage of solids-not-fat and found indications of the presence of mastitis; that is, high chlorine, low lactose, and high non-casein nitrogen.

The percentage of solids-not-fat in a sample taken from a single milking may differ from the daily average, but it varies less than the fat /Provan and Jenkins (139), Eckles and Shaw (54)/. Houston (87) found that the percentage of solids-not-fat in milk was nearly the same in afternoon and morning milkings when the over night period was 14 hours. Golding et al. (69) and Bartlett et al. (12) reported lower values for solids-not-fat in morning's milk. They determined solids-not-fat by fat and lactometer readings and conceded that the lactometer reading may have been taken before the milk from morning milkings had had time to reach its terminal value. Jones (101) found, on the other hand, that milk from the afternoon milkings was lower in solids-not-fat. He further observed that the difference was greatest in June, July, and August.

Environment of the Cow

The solids-not-fat content of milk is influenced by environmental conditions: Season, temperature, and feeding. The most notable effect of feeding is when the supply of nutrients is low, as under drought conditions during pasture season, so that the three factors tend to be associated in practice.

The effect of season on solids-not-fat content of milk seems to vary greatly from place to place, and from time to time. The highest solids-not-fat content usually occurs in the fall or early winter, October

through January. The low point may occur within a longer time period, April through September. A simple average of monthly average percentages of solids-not-fat reported by 12 investigators ranges from a high of 9.03 percent in January to a low of 8.84 percent in July (table 5).

Regan and Richardson (144) determined experimentally that the solids-not-fat percentage decreases with increasing temperature. The effect was greatest at temperatures above 80° F. Cows kept at 95° F. gave milk having about 0.7 percent lower solids-not-fat than cows kept at 50° F. Heinemann (78) correlated the solids-not-fat percentage of milk samples with daily temperatures. The correlation was closest during the summer months, a coefficient of -0.758 being found for June, when the percent of total solids in separated milk changed 0.119 percent with a temperature change of 1° F. In January the correlation coefficient reached a low of -0.062, and the regression coefficient was -0.0010. This finding is consistent with that of Regan and Richardson, to the effect that temperatures below 80° F. had less effect on the solids-not-fat percentage than did higher temperatures. Houston and Hale (88) correlated solids-not-fat yields with temperature, and found a partial correlation coefficient of -0.774 after accounting for variation in total milk yield. They considered temperature the equivalent of season.

Davis et al. (47) stressed temperature as a probable cause for low solids-not-fat in milk from cows in the vicinity of Tucson, Ariz. Schutte (167) mentions temperature as a factor in the solids-not-fat content of milk from cows in South Africa. As evidence that seasonal effects may be largely temperature or feeding, it was noted that the seasonal variation of temperature in South Africa is less than in northern countries, and that nutritional levels account for the seasonal high point in solids-not-fat being reached in October and November.

Season and temperature are closely related to drought as a factor influencing solids-not-fat, and drought in turn can be considered a matter of nutrient supply. Lloyd (116) reported that droughts of 1898 and 1899 caused milk of low solids-not-fat to be produced. He analyzed 19 samples in August 1898 and found an average of 8.27 percent solids-not-fat. In September, 34 samples averaged 8.41 percent. In the late 1920's and in the 1930's the effect of drought on solids-not-fat was noted in England, South Africa, New Zealand, and the United States [Cranfield (38), Nicholson and Lesser (129), Riddet et al. (157), Schutte (167), and Mathieson (121)].

Despite a general impression that the composition of milk is unaffected by the plane of nutrition of the cow, there is considerable evidence that underfeeding, as when feed supplies are reduced by drought, reduces the solids-not-fat content of milk. Nicholson and Lesser (129) attempted to increase the solids-not-fat of milk by the use of various supplements but without conclusive success. Davis et al. (49) observed that cows on marginal rations had essentially normal solids-not-fat. On the other hand, Rowland (161) and Riddet et al. (157) reported experiments in which reduced levels of nutrition were definitely accompanied by lowered

Table 5.—Solids-not-fat content of milk, by months, reported in various sources

Investigator	Year	Month											
		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Bakalor (8) 1/	1948	8.57	8.61	8.65	8.57	8.62	8.57	8.43	8.50	8.48	8.59	8.85	8.59
Baker and Cranfield (9)	1933	8.92	8.89	8.88	8.87	8.90	8.93	8.90	8.88	8.92	8.95	8.95	8.92
Cranfield (38)	1930	9.06	9.04	9.04	8.91	8.93	8.94	8.84	9.00	8.53	8.89	8.95	8.99
Davis et al. (49) 2/	1947	9.30	9.07	9.50	9.17	9.20	9.13	9.17	9.00	9.07	9.00	9.17	9.17
Jack et al. (94) 3/	1951	8.92	8.89	8.84	8.90	8.88	8.82	8.78	8.80	8.84	8.93	9.00	8.94
Jacobson (95) 4/	1936	9.00	9.00	9.00	9.01	8.98	9.09	8.96	8.90	8.98	9.07	9.02	8.92
Janse (98)	1950	8.80	8.75	8.82	8.80	8.91	8.84	8.77	8.75	8.97	8.99	8.89	8.80
Jones (101)	1935	8.95	8.94	8.93	8.88	8.91	8.89	8.78	8.76	8.85	8.92	8.95	8.94
Mathiesen (121)	1934	9.01	8.94	8.87	8.70	8.72	8.70	8.73	8.76	8.71	8.83	9.02	9.01
Shaw and Fourt (169)	1936	8.81	8.91	8.65	8.71	8.79	8.81	8.47	8.90	8.73	9.09	8.92	7.88
Sherman (171)	1906	9.37	9.39	9.27	9.18	9.17	9.11	8.96	9.02	9.15	9.26	9.35	9.43
Van Rensburg (147) 1/	1946	8.54	8.55	8.54	8.62	8.50	8.46	8.44	8.46	8.52	8.72	8.72	8.64
Vieth (188)	1889	9.18	9.19	9.17	9.13	9.13	9.14	9.06	9.04	9.12	9.20	9.15	9.10
White and Judkins (197)	1918	9.03	8.97	9.02	8.91	8.85	8.82	8.65	8.56	8.60	8.79	8.88	8.99
Average 5/		9.03	9.00	9.00	8.93	8.95	8.93	8.84	8.86	8.87	8.99	9.02	8.92

1/ This work was done in South Africa.
2/ Simple average of data for 3 breeds read from charts.
3/ Simple average of data for 2 years read from charts.
4/ Monthly values when annual average is 9.00 percent, based on Jacobson's chart of seasonal variation of solids-not-fat in milk containing 3.20 to 4.80 percent fat.
5/ Does not include data from Southern hemisphere.

percentages of solids-not-fat. Riddet et al. changed the rations of cows from full to half, in double-reversal experiments, and found that the solids-not-fat percentage was 0.3 to 0.5 percent lower while the lower plane of nutrition lasted. Rowland found that a low energy ration (75 percent of normal) with normal protein, and a low protein ration (60 percent of normal) with normal energy both resulted in lower solids-not-fat.

That feeding is a factor in the level of solids-not-fat is asserted also by Cranfield (37), Moore and Morrow (126), Provan and Jenkins (139), and Van Rensberg (147). Riddell (156) reviewed reports from various sources indicating that low levels of nutrition lowered the percentage of solids-not-fat in milk. Lesser (113) reported observations on one herd which had a subnormal percentage of solids-not-fat from November through March, becoming normal when the herd went on grass. Jones (101) commenting on a decline of solids-not-fat from March to April, which occurred in 9 years out of 10, ascribed it to excessive water consumption.

Stage of lactation has some influence on the level of solids-not-fat (see p. 18). It might thus be possible for stage of lactation to be a factor in the seasonal pattern of solids-not-fat percentages. White and Judkins (197) reported average percentages of solids-not-fat by months for lactations beginning in each month of the year. From these data, it is evident that under conditions in the herd at the Storrs, Conn., Experiment Station, seasonal variation was due largely to factors other than stage of lactation.

The conditions influencing the solids-not-fat content of milk also affect the fat percentage, and the effects are not always in the same direction or in the same proportions. A summary of quantitative relationships stated by various authors is given in table 6. On the basis of these data, an estimate of the solids-not-fat in the milk of an individual cow based on fat percentage alone might easily be 0.5 percent too high, or 0.6 percent too low. For example, in one sample out of 20 the fat percentage might be 0.08 percent or more too high, and solids-not-fat 0.12 percent or more too low, according to the precision of the Modified Babcock test for fat and the Mojonnier test for total solids as found by Heinemann et al. (78). At the end of the lactation period, the fat percentage averaged 0.44 percent over the lowest value, and solids-not-fat 0.17 percent higher in the data of White and Judkins (197) who also found that the first lactation has a higher ratio of fat to solids-not-fat than do later lactations, and that the highest ratio of fat to solids-not-fat occurs in May. As a result, with the variation due to analytical methods, a sample from a first calf heifer in the 10th month of lactation on a day in May averaging 95° might show an actual solids-not-fat percentage of about 8.6, instead of 9.2 percent normally accompanying a fat percentage of 4.92. Cumulative effects of opposite conditions--sampling in the third month of the third lactation, in September, with an air temperature of 80° might show an actual solids-not-fat of 9.2 instead of 8.7 percent normally accompanying a fat test of 3.66. The probability of deviations of these amounts cannot be stated. Yet they are based on average seasonal and lactation patterns. They exclude a great amount of random variation originally

Table 6.--Comparative influence of selected factors on fat and solids-not-fat of milk

Factor	Fat	Solids-not-fat
Method of analysis 1/	±0.042%	±0.060%
Breed 2/	2.56	1.52
Stage of lactation 3/	.44	.17
Age 4/	.24	.08
Season 3/	.31	.47
Temperature - High 5/	+ .3	- .3
Temperature - Low 6/	- .2	- .4

1/ Standard errors of the mean of 20 trials, Modified Babcock test and Mojonnier total solids test. Heinemann et al. (78).
2/ Range from lowest to highest values in table 3.
3/ Range of monthly averages from low month to high month, data of White and Judkins (197).
4/ White and Judkins (197), decline from 1st to 3d lactation.
5/ Regan and Richardson (144), change from 80° to 90° F.
6/ Change from 50° F. to 80° F.

present in the data from which the averages were determined. It is understandable, therefore, that various equations for estimating solids-not-fat from the fat percentage alone should show standard errors of estimate of the magnitude of 0.3 percent solids-not-fat.

Factors Influencing Specific Gravity

The possibility of increasing the precision of such estimating equations by including a specific gravity measurement depends on how constant is the specific gravity of the solids-not-fat. Explicitly or implicitly, a constant value is assumed for the specific gravity of solids-not-fat. This assumption has been variously treated by different writers, of which a few examples may be cited. Fleischmann (57) explicitly stated that the basic assumption was that the specific gravity of the solids-not-fat varied only within very narrow limits, and elsewhere (61), that owing to the small variations in the specific gravity of solids-not-fat, the formula would have only the status of a useful approximation, not absolute precision. Other writers were less exact; Pierre (135) wrote that fat-free milk has an invariable density of 1.6.

Misled, perhaps, by a tendency to disregard Fleischmann's reservations concerning the constancy of the specific gravity of solids-not-fat, de Waal (190, 191, 192, 193, 195) was sharply critical of formulas. He pointed out that protein, lactose, and ash have different specific gravities, and occur in different proportions in milk. However, he used this

fact only to deduce a different value for the specific gravity of skim milk than was used by Fleischmann, then committed the same error, if error it be, of considering specific gravity constant at the new value.

Jarl (100) showed that cows differed as individuals with respect to relationships between protein, lactose, and fat.

If variations in the specific gravity of solids-not-fat constitute an important source of error in estimating solids-not-fat from fat and specific gravity, then it might be of value to add another independent variable to the estimating equation. Koestler and Bakke (105, 106) noted that the solids-not-fat consisted of protein, lactose, ash, citric acid, and other substances having different specific gravities and present in varying proportions. De Waal (193) and Heinemann et al. (78) suggested equations including protein percentage as an independent variable. Inasmuch as milk protein has a specific gravity of about 1.35, as compared with 1.63 for lactose (168), and constitutes about 38 to 40 percent of the solids-not-fat (132), variations in the percentage of protein should be expected to explain much of the variation in specific gravity of the solids-not-fat. Heinemann et al. (78) found that use of the protein percentage reduced the standard error of estimate from 0.138 to 0.080 percent on a portion of their data.

If the fat and solids-not-fat percentages follow different patterns under the influence of season, temperature, lactation, age, and levels of nutrition it is important to know whether changes in the specific gravity of milk adequately account for deviations of solids-not-fat from its normal relationship to fat. The findings of Leonard (112) and Richmond (151) demonstrated a pronounced seasonal bias in the differences between gravimetric and lactometric measurements of fat which would be the reciprocal of similar measurements of solids-not-fat. Bartlett et al. (13) and Golding et al. (69) found similar patterns in comparing lactometric and gravimetric measurements of solids-not-fat, but the significance of their results was lessened by absence of control over the temperature of the milk samples between the time of milking and the time of taking the specific gravity. Bartlett et al. (13) quoted Salvestroni (164) as finding seasonal changes in the accuracy of results obtained by formulae. Boden and Campbell (18) found that lactometric methods gave low values during August through October and high values during February through April. The seasonal range was from 0.07 to 0.16 percent solids-not-fat in two groups of samples tested in duplicate at two laboratories. Their method was according to the British Standard Specification (24), which specifies heating to 40° C., then cooling and taking the lactometer reading at 68° F. Thus, an equation obtained by using a lactometer, and designed to give results that average the same as those obtained by the gravimetric method on a group of samples taken in equal numbers throughout the year, would be 0.035 to 0.08 percent too low at some seasons and too high by the same percentages at other seasons.

AIDS FOR COMPUTATION

Some work is involved in calculating the percentage of solids-not-fat from the fat percentage and specific gravity of milk. To make this as simple as possible, slide rules, tables, and nomographs have been devised. Fleischmann's formula was the basis for slide rules manufactured by Ackermann (65). Slide rules based on Richmond's formula also were available. Tables were published by Behrend and Morgen (15), Bohm (19), Fleischmann (57), Hawley (74), Hehner (75), Herz (82), Janse (99), Milk Industry Foundation (122), and Quesneville (140-142), from which total solids or solids-not-fat could be determined. These tables are based on various formulas.

Harris (72) gave a table for estimating fat from total solids and specific gravity, based on a transformation of Richmond's formula.

Horn (86) devised a lactometer reading total solids directly for 4.0 percent milk, subject to a correction of 1.2 percent total solids for each 1 percent of fat.

Nomograms (charts consisting of scales for fat percentage, specific gravity, and solids-not-fat percentage) have been published by Bycichin (29), Davis (45, 46), Lampert (108, 111) and Rueda (162).

MANUALS AND TEXTBOOKS

The literature on relationships among fat and solids-not-fat percentages and specific gravity of milk contains many citations to manuals, handbooks, or textbooks of various kinds. Some of these are cited so frequently, or have such authoritative standing that they should be noted in such a review as this. Among these, official and quasi-official manuals for the examination of foods and dairy products are most important. Codex Alimentarius Austriacus (33) relied on the Fleischmann formula. Codex Alimentarius Néerlandais (34) is a similar official manual using an independently derived formula (See table 1) which was sharply criticized by Van Dam (41, 42) and de Waal (190, 192). British Standard Specification No. 734 (24) establishes a procedure and formula, and describes hydrometers for determining the specific gravity and the solids-not-fat percentage of milk. Boden and Campbell (18) made a comparative evaluation of this method and the Richmond method.

Among the manuals and texts in which formulas are given for estimating solids-not-fat from fat and specific gravity, mention may be made of: Aufsberg (4), Barthel (11), Bockairy (17), Davies (44), Emmerich and Trillich (55), Fleischmann (58), Gerber (65), Hart and Tottingham (73), Henkel (79), Herz (83), Lampert (111), Leys (114), Lyons and O'Shea (118), Milk Industry Foundation (122), Möllgaard (124), Richmond et al. (155), Roadhouse and Henderson (158), Sommer (173, 174), Teichert (177), and Wiley (198).

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